Cornell University®

Big Red Math Competition
Individual Round

Name: _____
Student ID: _____
October 2026 11:15am–01:00pm

INSTRUCTIONS — PLEASE READ THIS NOW

- **Write your Name and Student ID.**
- **Simplification.** Please simplify all answers as much as possible. For example, $\frac{\cos \pi}{\frac{1}{2}}$ can be simplified to -2.
- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources.
- All problems are worth the same in this round.
- **You have 105 minutes to complete this exam.**
- **Note.** The problems may be easier than they first appear. Do not be intimidated by them. Just start with what you can do and build from there. Good luck!

Please sign below to indicate that you have read and agree to these instructions.

Signature, Print Name: _____

Answer Sheet:

1. _____	5. _____	9. _____	13. _____
2. _____	6. _____	10. _____	14. _____
3. _____	7. _____	11. _____	15. _____
4. _____	8. _____	12. _____	

Please don't mark anything below on the page, it's for official use only.

Grader 1 Name:	Grader 2 Name:	Grader 3 Name:
_____	_____	_____
Tally:	Tally:	Tally:
_____	_____	_____
Score:	Score:	Score:
_____	_____	_____

1. Find the coefficient of $x^4y^7z^4$ in the expansion of

$$(x^2 - 3y^3 + 4y^2z^2)^5,$$

after all like terms are collected.

2. A quadrilateral $ABCD$ has interior angles $\angle ABC = 120^\circ$ and $\angle BCD = 90^\circ$. A circle is tangent to segments AB and CD, and is also tangent to segment BC at the point P. Compute BP/CP.

3. Let x be a real number with $x \neq 0, 1$. Define

$$y = 1 - \frac{1}{x} \qquad \text{and} \qquad z = 1 - \frac{1}{y}.$$

Suppose $x + y + z = 7$. Find $x^2 + y^2 + z^2$.

4. For each positive integer n, let

$$d_n = \gcd(n^2 + n + 9, n^2 + 3n + 15).$$

Find $\displaystyle\sum_{n=1}^{2026} d_n$.

5. What is the largest value of n for which it is possible to arrange n distinct positive integers, each less than or equal to 20, around a circle so that any two adjacent numbers have greatest common divisor greater than 1?

6. The real numbers c, o, r, n, e, ℓ satisfy

$$c + o + r + n + e + \ell + \ell = 20$$

and

$$(c-1)^2 + (o-1)^2 + (r-1)^2 + (n-1)^2 + (e-1)^2 + 2(\ell - 1)^2 = 37.$$

Determine the maximum possible value of ℓ.

7. Let $P(x)$ be a polynomial with integer coefficients satisfying $P(1) = 1$ and $P(3) = 5$. Find the smallest possible value of $|P(7)|$.

8. Square $ABCD$ has side length 12. Points E and F lie on sides BC and CD, respectively, such that $\triangle AEF$ is equilateral. Lines BF and DE intersect at P. Find AP^2.

9. Suppose that a, b, c are positive integers satisfying

$$\begin{aligned} a\gcd(b, c) &= 1200, \\ b\gcd(c, a) &= 432, \\ c\gcd(a, b) &= 1344. \end{aligned}$$

Find the greatest possible value of $\operatorname{lcm}(a, b, c)$.

10. A 5-element subset S of $\{0, 1, \ldots, 11\}$ is called *spaced* if no two elements of S are consecutive modulo 12. For example, $\{0, 2, 4, 6, 8\}$ is spaced, while $\{0, 2, 4, 6, 11\}$ is not. For each spaced subset S, define

$$F(S) = \left(\sum_{k \in S} \cos \frac{k\pi}{6}\right)^2 + \left(\sum_{k \in S} \sin \frac{k\pi}{6}\right)^2.$$

How many spaced subsets S satisfy $F(S) < \frac{1}{2}$?

11. Let s be a positive integer, and let $ABCD$ be a square of side length s. Suppose that there is at least one point P strictly inside the circumcircle of $ABCD$ with $AP = 20$ and $CP = 40$, and that every such point P lies outside the square. Find the sum of all possible values of s.

12. Aryaan wants to place the numbers $1, 2, \ldots, 200$ in the cells of a 2×100 grid, one number per cell, so that each pair of consecutive numbers lies in cells that share a side. In how many ways can Aryaan accomplish this task?

13. An integer x_0 is chosen uniformly at random from $\{1, 2, \ldots, 2025\}$. For each integer $j \geq 0$, define x_{j+1} to be the least positive residue of $45x_j$ modulo 2026. Suppose the probability that $x_0 < x_1 < x_2$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

14. For each nonzero complex number z, define the bidirectional geometric sequence $S_z = (a_n)_{n \in \mathbb{Z}}$ by

$$a_0 = z, \qquad a_{n+1} = (1+i)a_n \quad \text{for all integers } n.$$

For example, S_1 contains the values

$$1, \quad 1+i, \quad 2i, \quad -2+2i, \quad -4$$

at indices $0, 1, 2, 3, 4$, respectively. Moreover, S_{1+i} contains exactly the same values as S_1, but at different indices.

There exists a set of nonzero complex numbers $\{z_1, z_2, \ldots, z_k\}$ such that, for all integers a, b satisfying $1 \leq a^2 + b^2 \leq 36$, the complex number $a + bi$ occurs in at least one of $S_{z_1}, S_{z_2}, \ldots, S_{z_k}$. Compute the smallest positive integer k for which such a set exists.

15. For a real number x, define a sequence $a_0, a_1, \ldots$ by

$$a_0 = x, \qquad a_{n+1} = a_n^2 - 2 \quad (n \geq 0).$$

Let S be the set of all real numbers x such that $a_5 = a_0$ but $a_1 \neq a_0$. Find $\sum_{x \in S} x^2$.