



Cornell University®

Big Red Math Competition
Proof Round

Name: _____

Student ID: _____

Oct 24th, 2026

9:30am–11:00am

INSTRUCTIONS — PLEASE READ THIS NOW

- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources.
- **Show your work.** To receive full credit, your answers must be neatly written and logically organized.
- **Simplification.** Please simplify all answers as much as possible. For example, $\frac{\cos \pi}{\frac{1}{2}}$ can be simplified to -2 .
- **You have 90 minutes to complete this exam.**
- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources.
- **Note.** The problems may be easier than they first appear. Do not be intimidated by them. Just start with what you can do and build from there. Good luck!

Please sign below to indicate that you have read and agree to these instructions.

Signature of Student: _____

Please don't mark anything below on the page, it's for official use only.

Grader 1: _____	Grader 2: _____	Grader 3: _____
1. _____ / 15 _____	1. _____ / 15 _____	1. _____ / 15 _____
2. _____ / 15 _____	2. _____ / 15 _____	2. _____ / 15 _____
3. _____ / 25 _____	3. _____ / 25 _____	3. _____ / 25 _____
4. _____ / 25 _____	4. _____ / 25 _____	4. _____ / 25 _____
Total: _____ / 80 _____	Total: _____ / 80 _____	Total: _____ / 80 _____

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1. A nonempty finite set A of positive integers is called *quadratic* if the sum of its elements equals the square of the number of its elements. For example, $\{1, 2, 3, 10\}$ is quadratic, since it is a set with 4 elements and $1 + 2 + 3 + 10 = 4^2$. Recall that the elements of a set must be distinct.
 - (a) Give an example of a quadratic set with 2026 elements.
 - (b) Prove that every quadratic set contains at least one odd element.
 - (c) Prove that any two quadratic sets with the same number of elements must have at least one element in common.

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2. Let a , b , and c be positive integers. Determine all triples (a, b, c) for which

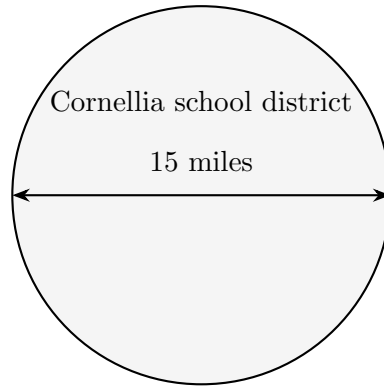
$$(b^2 - c^2)^2 = a^2 + b + 3.$$

Justify your answer.

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3. The Cornellia school district is a circular region 15 miles in diameter. Students may live anywhere in the district, and elementary schools may be built anywhere in it, including on its boundary.

The district wants every student to live within 5 miles of at least one school. Distances are measured in straight lines.



School locations are to be chosen.

- (a) Show how to place 8 elementary schools so that every student lives within 5 miles of a school.
- (b) Show that 6 elementary schools suffice to provide the same guarantee.
- (c) What is the smallest number of elementary schools needed to provide this guarantee? Exhibit a placement that works for this number of schools and prove that fewer schools cannot suffice. Depending on your approach, you may find it helpful to know $\cos 36^\circ = \frac{1+\sqrt{5}}{4}$.

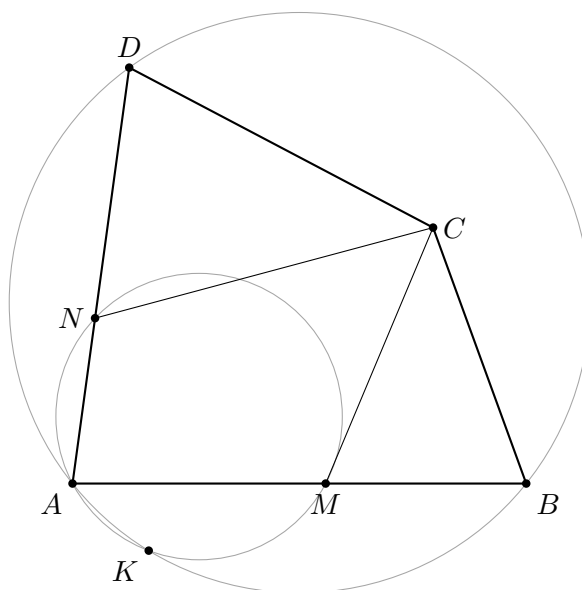
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4. Let $ABCD$ be a convex quadrilateral with $\angle ABC = \angle ADC$. Let M and N lie in the interiors of sides AB and AD , respectively, such that $\angle MCD = \angle NCB$.

Suppose that the circumcircles of triangles AMN and ABD intersect at A and at a second point $K \neq A$. Prove that $AK \perp CK$.



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