



Cornell University®

Big Red Math Competition
Team Round

Team ID: _____
October 24th, 2026 2:05pm–
2:35pm

INSTRUCTIONS — PLEASE READ THIS NOW

- **Write your Team ID. ONLY captain needs to turn in.**
- **Simplification.** Please simplify all answers as much as possible. For example, $\frac{\cos \pi}{\frac{1}{2}}$ can be simplified to -2 .
- **You have 30 minutes to complete this exam.**
- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources. You **CAN** collaborate with your teammates.
- **Note.** The problems may be easier than they first appear. Do not be intimidated by them. Just start with what you can do and build from there. Good luck!

Please sign below to indicate that you have read and agree to these instructions.

Signature of Captain, Print Name: _____

Answer Sheet:

- | | | |
|----------|----------|-----------|
| 1. _____ | 5. _____ | 9. _____ |
| 2. _____ | 6. _____ | 10. _____ |
| 3. _____ | 7. _____ | |
| 4. _____ | 8. _____ | |
-

Please don't mark anything below on the page, it's for official use only.

Grader 1 Name:

Grader 2 Name:

Grader 3 Name:

Tally:

Tally:

Tally:

Score:

Score:

Score:

1. The diagonals of convex quadrilateral $ABCD$ intersect at E . Suppose that $AE = BE = CE = CD$ and $\angle ECD = 36^\circ$. If $\alpha = \angle ABE$ and $\beta = \angle BCE$, find α/β .
2. A Cornell professor's teaching assistants (TAs) belong to exactly one of two categories: undergraduate TAs or graduate TAs. The professor wants to choose three distinct TAs to proctor an exam. The group must consist of either two undergraduate TAs and one graduate TA, or two graduate TAs and one undergraduate TA.

There are exactly 63 groups satisfying this requirement. Groups are distinguished only by their members; the order of selection does not matter. How many TAs does the professor have in total?

3. Let S be the set of all real numbers θ with $0 \leq \theta < 2\pi$ satisfying

$$\sin \theta + 2 \sin(2\theta) + 3 \sin(3\theta) = 0.$$

Find $\sum_{\theta \in S} \frac{1}{\cos^2 \theta}$.

4. Let AB be a diameter of a unit circle ω_1 centered at O . Circles ω_2 and ω_3 lie inside ω_1 and are each tangent to both ω_1 and AB . Suppose that both O and the center of ω_3 lie on ω_2 . Find the radius of ω_3 .
5. Compute the exponent of 2 in the prime factorization of

$$\prod_{k=1}^{342} (3k - 2) = 1 \cdot 4 \cdot 7 \cdots 1024.$$

6. Let $a_1, a_2, \dots, a_{10}$ be positive real numbers satisfying $a_1 a_2 \cdots a_{10} = 1$. Find the minimum possible value of

$$a_1 + a_2^2 + a_3^2 + a_4^3 + a_5^3 + a_6^3 + a_7^4 + a_8^4 + a_9^4 + a_{10}^4.$$

7. Call an integer $n \geq 2$ *Ithacan* if, for every nonnegative integer k such that $2^{k+2} \leq n$,

$$\binom{n}{2^k} + \binom{n}{2^{k+1}} + \binom{n}{2^{k+2}} \equiv 1 \pmod{2}.$$

For example, 4 is Ithacan, since the only possible value of k is 0, and

$$\binom{4}{1} + \binom{4}{2} + \binom{4}{4} = 4 + 6 + 1 = 11 \equiv 1 \pmod{2}.$$

Find the sum of all Ithacan integers less than or equal to 63.

8. A necklace consists of 2026 blue beads and some red beads, arranged in a circle. Every block of consecutive beads that contains exactly 8 blue beads contains at least 5 red beads. What is the smallest possible number of red beads on the necklace?
9. Determine the smallest integer $n \geq 4$ such that, among any n distinct positive integers relatively prime to 13, one can choose four pairwise distinct numbers a, b, c, d satisfying

$$13 \mid ad - bc.$$

10. Two congruent trapezoids, each having interior angles of measures 23° and 34° , are placed in the plane so that their boundaries divide the plane into regions, three of which are bounded regions that are mutually similar triangles. Let θ be the degree measure of the largest angle of these triangles. Compute the sum of all possible values of θ .